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Complex method determination of heat conductivity and thermal diffusivity coefficients

Summary

The present work is focused on the development of experimental method determination of heat conductivity λ and thermal diffusivity coefficients of food products and other materials. The methods determinations of coefficients based on a theoretical foundations and mathematical dependencies of the theory and method of regular mode and its further development. The way of determining thermal diffusivity coefficient is shown. It does not need a condition when surface heat transfer α is near to infinitely, which is a premise of the regular mode method. The method of complex experimental determination of heat conductivity and thermal diffusivity coefficients by means of one experimental plant with only two tests was described.

Key words: *regular mode, food products, heat conductivity, thermal diffusivity coefficient, cod meat*

Kompleksowa metoda wyznaczania współczynnika przewodzenia ciepła i dyfuzyjności cieplnej materiałów

Streszczenie

W pracy skoncentrowano się na przedstawieniu rozwoju eksperymentalnej metody wyznaczania współczynnika przewodzenia ciepła λ i dyfuzyjności cieplnej a w produktach żywnościowych i innych materiałach. Metody wyznaczania w/w współczynników bazują na teoretycznych podstawach i matematycznych zależnościach teorii stanu uporządkowanego i podlegają cały czas dalszemu rozwojowi. W przedstawionej metodzie wyznaczania dyfuzyjności cieplnej nie musi być spełniony warunek, aby współczynnik wnikania ciepła dążył do nieskończoności, co jest warunkiem koniecznym w metodzie stanu uporządkowanego.

Słowa kluczowe: *metoda stanu uporządkowanego, produkty żywnościowe, współczynnik przewodzenia ciepła, dyfuzyjność cieplna, mięso dorsza*

Introduction

Intensiveness and energy consumption for production and storage of food products processes are complex in structure and chemical composition systems that determine to a large extent their thermophysical characteristics (TC). They are used in calculating durations and the processes optimizing, heat loads, equipment designing and creating products models (Aerlichman et al., 2014).

Thermophysical characteristics (TC) of different products and chemical composition were investigated by many scientists (Jason et al., 1955; Riedel, 1956; Hill, 1967; Ginzburg et al., 1975; Marcotte et al., 2008) and others. But they do not cover the whole range of raw and products manufactured within a wide range of temperatures. Besides, an intensive development of new functional products for various age and professional population groups. It determines vitality of developing new methods and continuation of work for defining their TC.

The definition of thermal characteristics presents certain difficulties due to their heterogeneity, high labor consumption and complexity of experimental sets. Special difficulties are encountered while defining systems TC of systems which are composed of separate products and placed into one container, e.g. in the form of fish or meat blocks.

Substantial difficulties occur when defining TC of products in a frozen state, because their TC changes, depending on the temperature due to turning moisture into ice.

Such TC as specific heat capacity c and density ρ are additive values and may be defined for the food products by calculation method on the known mass and chemical product composition of the products.

Heat conductivity coefficients λ and a are not such as that and they are defined exclusively in an experimental way (Dickerson, 1965; Dua and Oyna, 1969; Simson and Cortes, 2004; Platunov et al., 2010). The simplest and the most reliable method of determining thermal diffusivity coefficient a, is suggested by Kondratiev (Kondratiev, 1957). This method is based on the heat conductivity theory and it allows a rather simple way of defining the thermal diffusivity coefficient in the process of cooling or heating.

$$\alpha = \frac{\lambda}{c\rho} \tag{1}$$

However, its application is valid on the condition that surface heat transfer coefficient from the product sample to a medium during refrigeration or from the medium to the products during heat transfer tends to infinity ($\alpha \to \infty$). Additionally, the low thermal conductivity coefficient results in that the Biot number also tends to infinity.

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$$Bi = \frac{\alpha \cdot l}{\lambda} \quad \textbf{(2)}$$

(in practice Bi > 100). In the case of slabs having a thickness dimension of the linear characteristic

$$l = \frac{\delta}{2} \quad \textbf{(3)}$$

It means that constant predetermined temperature "t_0" should be kept on the outer surface "F" of the sample being investigated and marginal condition of the third order is secured.

On this condition the graph of excessive temperature logarithm change on time $\ln(t - t_0) = f(\tau)$, where t is temperature within the sample, presents a direct line, and the thermal diffusivity coefficient is calculated according to the formula:

$$a = Km_\infty \quad \textbf{(4)}$$

where:
- K – sample form coefficient, m²;
- m_∞ – sample cooling rate at $\alpha \to \infty$, and equals:

$$m_\infty = \frac{\ln(t_1 - t_0) - \ln(t_2 - t_0)}{\tau_2 - \tau_1} \quad \textbf{(5)}$$

where:
- t_1 and t_2 – temperature in any point of sample at time moments τ_1 and τ_2 [°C].

Condition $\alpha \to \infty$ may be obtained only to a certain precision. Selection of α value when using the regular mode method was developed by Kondratiev (1957) and is based on a universal dependence between criterion M and modified criterion H, described by the formulas

$$M = \frac{m}{m_\infty} \quad \textbf{(6)}$$

$$H = \frac{\alpha}{\lambda}\frac{KF}{V} \quad \textbf{(7)}$$

where:
- m – sample cooling rate at final value of α;
- V – sample volume, [m³];
- F – heat exchange surface, [m²].

It follows from formula (7) that surface heat transfer coefficient equals:

$$\alpha = H\frac{\lambda V}{KF} \quad \textbf{(8)}$$

The relation between criteria H and M may be taken from the detailed table which is given in the paper of Kondratiev, excerpts of which are presented below:

Table 1. Relation between criteria H and M

Tabela 1. Zależność pomiędzy kryterium H i M

H	∞	25	20	15	10	0
M	1	0.972	0.965	0.954	0.931	0

At H = 15 criterion M is other than one, which corresponds to an ideal condition $H \to \infty$ and $\alpha \to \infty$, at $\frac{1-0.954}{1} \cdot 100\% = 4.6\%$. We take an error due to non observance of condition $\alpha \to \infty$ 4.6% then at $H \geq 15$, or as follows from equation (8) at $\alpha \geq 15\lambda V/KF$ we may consider $M \approx 1$ and take $\alpha \to \infty$ with the error 4.6%. In practice, when carrying out experiments, the condition $\alpha \to \infty$ is secured by way of high speed medium flushing the sample being investigated either at boiling or condensing, or by selecting sample dimensions defining values K, F and V, which follows from equation (8).

The main advantages of the experimental methods based on the theory of regular mode in heat transfer are short time and simple measurements with sufficient precision (Wiśniewski, 2000).

The aim of the work

The aim of the work was to present the possibility of applying a modified method, based on the theory of regular mode to determine the thermal conductivity and thermal diffusivity of food products for example meat cod.

Material and method

In a number of cases it is not possible to provide condition $\alpha \to \infty$, or the graph of dependence of excessive temperature logarithm on time $\ln(t - t_0) = f(\tau)$ presents a curve concaved upward. It takes place at defining the thermal diffusivity coefficient of products in the frozen state, when we have TC with the sample temperature changes.

In the case of impossibility to create conditions $\alpha \to \infty$ with the predetermined accuracy, it is possible to determine coefficients of thermal diffusivity and heat conductivity of products with the known specific product thermal capacity based on dependencies underlying the regular method mode. For that purpose cooling rate "m" is determined from graph $\ln(t - t_0) = f(\tau)$, obtained experimentally at the final value of surface heat transfer coefficient α_K, which is considerably different from the condition $\alpha \to \infty$. Then thermal diffusivity coefficient a_0 is determined by expression (4) and heat conductivity coefficient $\lambda_0 = a_0 c\rho$ is determined by equation (1). By means of dependency (7) criterion H is determined:

$$H = \frac{\alpha_K}{\lambda_0}\frac{KF}{V} \quad \textbf{(9)}$$

by means of which criterion M is determined from the table of Kondratiev. The M value can be determined by equation

$$M = \frac{H}{\sqrt{H^2 + 1.437H + 1}} \quad (10)$$

Then the true value of sample cooling rate at $\alpha \to \infty$ will be equal to:

$$m_\infty = \frac{m}{M} \quad (11)$$

The demonstrated experimental method allows to determine only thermal diffusivity coefficient a on formula (4). The heat conductivity coefficient λ can be determined, if is known specific heat capacity c and density ρ. For the experimental determination of heat conductivity λ by method of regular mode or any other method the second experimental plant is required, which makes research more complex and expensive. By way of developing the presented experimentally – calculated complex method has been developed, which allows to determine heat conductivity, thermal diffusivity coefficients and specific heat capacity of the material. This method do not need creating conditions when heat transfer coefficient $\alpha \to \infty$ to. The method is based on criteria M (formula 6), H (formula 7) and relation between them (formula 10). It is not difficult to obtain expressions for sample cooling rate at $\alpha \to \infty$.

$$m_\infty = m \frac{\sqrt{\left(\frac{\alpha}{\lambda} A\right)^2 + 1.437 \frac{\alpha}{\lambda} A + 1}}{\frac{\alpha}{\lambda} A} \quad (12)$$

where: $A = \frac{KF}{V}$.

It follows from formula (12) that from a test performed at any final value of surface heat transfer coefficient $\alpha < \alpha \to \infty$, it is not possible to determine refrigeration rate m_∞ if we don't know material heat transfer coefficient λ.

Sample cooling rate m_∞ and also material heat conductivity coefficient can be determined if two tests are performed at two different and final values of heat transfer coefficients α_1 and α_2. Having determined refrigeration rates m_1 and m_2 by means of tests results corresponding to α_1 and α_2 it is possible to set up and solve a system of two equations:

$$m_\infty = m_1 \frac{\sqrt{\left(\frac{\alpha_1}{\lambda} A\right)^2 + 1.437 \frac{\alpha_1}{\lambda} A + 1}}{\frac{\alpha_1}{\lambda} A} \quad (13)$$

$$m_\infty = m_2 \frac{\sqrt{\left(\frac{\alpha_2}{\lambda} A\right)^2 + 1.437 \frac{\alpha_2}{\lambda} A + 1}}{\frac{\alpha_2}{\lambda} A} \quad (14)$$

Solution of equations (13) and (14) in relation to heat conductivity coefficient has the form:

$$\lambda = \frac{1}{-\frac{0.718(\alpha_1 B^2 - \alpha_2)}{A(\alpha_1^2 B^2 - \alpha_2^2)} + \sqrt{\left(\frac{0.718(\alpha_1 B^2 - \alpha_2)}{A(\alpha_1^2 B^2 - \alpha_2^2)}\right)^2 - \frac{B^2 - 1}{A^2(\alpha_1^2 B^2 - \alpha_2^2)}}} \quad (15)$$

where:

$$B = \frac{m_1}{m_2}\frac{\alpha_2}{\alpha_1} \quad (16)$$

Applying formula (13) or (14) it is possible to determine sample cooling rate m_∞ and thermal diffusivity coefficient a by means of formula (4). Heat transfer coefficient α at performing tests is determined by a heat flux sensor q using the method given in the work (Platunov et. al., 2010) and by the results of measuring sample surface temperature t_n and refrigeration medium t_0

$$\alpha = \frac{q}{t_n - t_0} \quad (17)$$

At $\alpha < \alpha \to \infty$, the surface temperature in experiments is not equal to temperature of refrigeration medium $t_n \neq t_0$. So a mean integral value ā is used in calculations which is determined by graphic integrating an experimental dependence curve $\alpha = f(t_n - t_0)$. In the processes with the change of phase (freezing, defrosting) when heat conductivity coefficient depends a great deal on temperature, dependence $\ln(t - t_0) = f(\tau)$ presents a convex upward curve. In this case it is necessary to act likewise but within a narrow temperature range. Testing the two proposed techniques (methods) was carried out on the experimental plant the layout of which is shown on Figure 1.

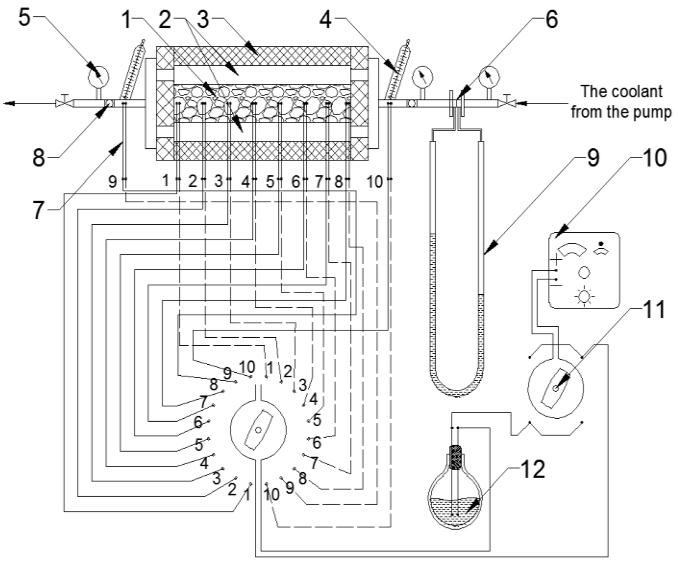

Fig. 1. Experimental unit chart: 1 – investigated material; 2 – refrigeration plates; 3 – heat insulator; 4 – thermometer; 5 – manometer; 6 – chamber diaphragm; 7 – thermo-couple; 8 – observation glass; 9 – differential manometer; 10 – potentiometer; 11 – switch; 12 – Dewar vessel

Rys. 1. Schemat stanowiska badawczego: 1 – badany materiał; 2 – płyty chłodnicze; 3 – izolacja cieplna; 4 – termometr; 5 – manometr; 6 – kryza pomiarowa; 7 – termopara; 8 – szkło obserwacyjne; 9 – manometr różnicowy; 10 – potencjometr; 11 – przełącznik; 11 – naczynie Dewara

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Experimental plant allowed regulation of coolant quantity to the plates and its passage through two the ducts in the super cooled state, which made it possible to carry out the experiment at different heat transfer coefficient and to define their value by the calculation method. The coolant was Freon R22, which was pumped into the ducts. The temperature of the coolant was t_0 = 0°C and was generated by the refrigerating plant. Paraffin was applied in experiments with the first method and fish mince of cod meat with the second method. Paraffin and fish mince of cod meat in form of parallelepiped with dimensions 0.598 × 0.255 [m] and thickness δ = 0.07 [m] is placed between two plates having passages for coolant.

So the surface of heat removal, block form coefficient, its volume and a coefficient were equal to

$$F = 2 \cdot 0.598 \cdot 0.255 = 0.305 [\mathrm{m}^2],$$

$$K = \frac{\delta^2}{\pi^2} = \frac{0.07^2}{3.14^2} = 4.97 \cdot 10^{-4} [\mathrm{m}^2],$$

$$V = 0.598 \cdot 0.255 \cdot 0.07 = 1.067 \cdot 10^{-2} [\mathrm{m}^3],$$

and

$$A = \frac{KF}{V} = \frac{4.97 \cdot 10^{-4} \cdot 0.305}{1.067 \cdot 10^{-2}} = 1.421 \cdot 10^{-2}.$$

For the experimental paraffin sample, its volume, mass, density and surface heat transfer coefficient were according m_n = 9.23 [kg], ρ = 865 [kgm^{-3}] and α_1 = 686 [Wm^{-2}K^{-1}] and α_2 = 150 [Wm^{-2}K^{-1}]

For the purpose of assessment of the obtained experimental results validity, the TCs of paraffin were used, obtained from different sources, and they differed considerably. Thus some papers give out the following paraffin TC: α = 1.25·10^{-7} [m^{2}s^{-1}], λ = 0.1465 [Wm^{-1}K^{-1}], c = 1.37 [kJkg^{-1}·K^{-1}] and ρ = 850 [kgm^{-3}]. While some other papers give data for specific thermal capacity and thermal diffusivity coefficient of paraffin at t = +20°C, λ = 0.27 [Wm^{-1}K^{-1}], ρ = 920 [kgm^{-3}]. The density of minced block of cod meat under tentative pressure with the plates of p = 0,005 MPa was ρ = 950 [kgm^{-3}]. Experiments for determining thermal diffusivity coefficient and heat conductivity coefficient were made at surface heat transfer coefficients α_1 = 160 [Wm^{-2}K^{-1}] and α_2 = 350 [Wm^{-2}K^{-1}].

Results and discussion

The results of the experiments for paraffin and at two surface heat transfer coefficients values α = 686 [Wm^{-2}K^{-1}] and α = 150 [Wm^{-2}K^{-1}] at specific heat capacity c = 1.37 [kJkg^{-1}·K^{-1}] are given in table 2.

Table 2. The experiments results

Tabela 1. Wyniki eksperymentów

Parameters; Paramery	**Heat transfer coefficient; Współczynnik wnikania ciepła α [Wm^{-2}K^{-1}]**	
	686	**150**
Cooling rate; Tempo chłodzenia $m \cdot 10^4$ [s^{-1}]	2.85	2.75
Thermal diffusivity coefficient; Dyfuzyjność cieplna $\alpha_0 = Km \cdot 10^7$ [m^2 s^{-1}]	1.416	1.367
Heat conductivity coefficient; Współczynnik przewodzenia ciepła $\lambda_0 = \alpha_0 c\rho$ [Wm^{-1} K^{-1}]	0.168	0.162
Criterion H; Kryterium H	58	13.2
Criterion M; Kryterium M	0.987	0.947
Error M on condition $\alpha \to \infty$; Błąd M przy warunku $\alpha \to \infty$ $\frac{1-M}{M} \cdot 100\%$	1.3	5.6
Cooling rate at $\alpha \to \infty$; Tempo chłodzenia przy $\alpha \to \infty$; $m_\infty = \frac{m}{M} \cdot 10^4$ [s^{-1}]	2.89	2.90
Thermal diffusivity coefficient; Dyfuzyjność cieplna $\alpha_0 = Km_\infty \cdot 10^7$ [m^2 s^{-1}]	1.43	1.44

The demonstrated experimental-calculation method allows to determine thermal diffusivity coefficient by regular mode method even in the case of impossibility to observe condition of $\alpha \to \infty$. The difference in determination of thermal diffusivity coefficients is less than 1% at α = 686 [Wm^{-2}K^{-1}] and α = 150 [Wm^{-2}K^{-1}]

The final thermal diffusivity coefficient and heat conductivity coefficient of paraffin will be obtained.

$$\alpha = Km_\infty = 4.97 \cdot 10^{-4} \cdot 2.90 \cdot 10^{-4} = 1.44 \cdot 10^{-7} [\mathrm{m}^2 \mathrm{s}^{-1}]$$

$$\lambda = \alpha c\rho = 1.44 \cdot 10^{-7} \cdot 1.37 \cdot 10^{3} \cdot 865 = 1.17 [\mathrm{Wm}^{-1}\mathrm{K}^{-1}]$$

At that heat conductivity coefficient can be determined and, prior to that, the specific thermal capacity was obtained, e.g. by calorimetric method. As the result of tests for refrigerating block form of cod meat were obtained the following results: at α_1 = 160 [Wm^{-2}K^{-1}], m_1 = 0.166·10^{-3} [s^{-1}] and at α_2 = 350 [Wm^{-2}K^{-1}] m_2 = 0.175·10^{-2} [s^{-1}]. Then according to formula (16)

$$B = \frac{m_1}{m_2} \frac{\alpha_2}{\alpha_1} = \frac{0.166 \cdot 10^{-3}}{0.175 \cdot 10^{-2}} \cdot \frac{350}{160} = 2.075$$

and according to formula (15) heat conductivity coefficient

$$\lambda = \frac{0.718(150 \cdot 2.075^2 - 350)}{1.421 \cdot 10^{-2}(150^2 \cdot 2.075^2 - 350^2)} +$$
$$+ \sqrt{\left(\frac{0.718(150 \cdot 2.075^2 - 350)}{1.421 \cdot 10^{-2}(150^2 \cdot 2.075^2 - 350^2)}\right)^2 - \frac{2.075^2 - 1}{1.421 \cdot 10^{-4}(150^2 \cdot 2.075^2 - 350^2)}} =$$
$$= 0.312 \left[\mathrm{Wm^{-1}}K^{-1}\right]$$

Making use of this regulation it is not difficult to obtain value (13) of sample cooling rate at $\alpha \to \infty$

$$m_\infty = 0.166 \cdot 10^{-3} \frac{\sqrt{\left(\frac{160}{0.312} \cdot 1.421 \cdot 10^{-2}\right)^2 + 1.437 \frac{160}{0.312} \cdot 1.421 \cdot 10^{-2} + 1}}{\frac{160}{0.312} \cdot 1.421 \cdot 10^{-2}} =$$
$$= 0.183 \cdot 10^{-3} \left[\mathrm{s^{-1}}\right]$$

and making use of formula (4) to calculate thermal diffusivity coefficient

$$\alpha = Km_\infty = 4.97 \cdot 10^{-4} \cdot 0.184 \cdot 10^{-3} = 0.916 \, [\mathrm{m^2 \, s^{-1}}].$$

As the block density of non-frozen mince ρ = 950 [kgm^{-3}], then its specific thermal capacity

$$B = \frac{\lambda}{\alpha\rho} = \frac{0.312}{0.916 \cdot 10^{-7} \cdot 950} = 3585 \left[\mathrm{kJkg^{-1}K^{-1}}\right]$$

For experimentally determined thermal conductivity coefficient λ = 0.318 [Wm^{-1}K^{-1}] the criterion value H at heat removal coefficient α_1 = 160 [Wm^{-2}K^{-1}].

$$H = \frac{\alpha_1}{\lambda} A = \frac{160}{0.312} \cdot 1.421 \cdot 10^{-3} \approx 7.3$$

According to the table of criteria dependence $M = f(H)$ at $H = 7.3$; $M = 0.906$.

On the other hand at α_1 = 160 [Wm^{-2}K^{-1}] sample cooling rate was $m_1 = 0.166 \cdot 10^{-3}$ [s^{-1}] and then according to formula (6):

$$H = \frac{m}{m_\infty} = \frac{0.166 \cdot 10^{-3}}{0.183 \cdot 10^{-3}} = 0.907$$

Practical agreement of criteria M gives evidence for correctness of the method suggested.

Conclusions

1. The methods suggested are the development of regular mode method and allows determining coefficients of heat conductivity and thermal diffusivity by means of one experimental plant.
2. The methods features simplicity and more accurateness, as it doesn't need creating condition of making an experiment at $\alpha \to \infty$.
3. If material density is known and determination of which is not too difficult, its specific thermal capacity is easy to determine by a calculation method making use of values of heat conductivity and thermal diffusivity coefficients.

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